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Type I Superconductivity of Protons in Neutron Stars

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ABSTRACT
The magnetic structure of neutron vortices in the superfluid cores of neutron stars is determined assuming that the proton condensate forms a type I superconductor. It is shown that the entrainment currents induced by the neutron vortex circulation cause the proton superconductor to break into successive domains of normal and superconducting regions. The Gibbs free-energy is found in the case in which the normal domains form cylindrical tubes coaxial with the neutron vortex. The minimum of the energy functional corresponds to a tube radius $a \sim 0.1 - 0.5\ b$, where b is the outer radius of the neutron vortex. The magnetic field within the tube is of the order of 5×10^{14} G.

Key words: MHD – stars: neutron – pulsars: general – stars: rotation.

1 INTRODUCTION

The knowledge of the properties of neutron vortex lattice in the quantum liquid cores of a pulsar (in the simplest case a mixture of neutrons, protons, and electrons, called further n-p-e phase) is important for the understanding of the coupling mechanisms between the superfluid and normal component of the star. The problem differs from the case of vortices in a rotating uncharged superfluid mainly due to the fact that above the nuclear saturation density, $\rho_{\rm sat} \simeq 2.6 \times 10^{14}$ g cm^{-3}, the $^3P_2$ superfluid neutron vortex lattice is embedded in a $^1S_0$ proton superconductor. Under these conditions, the relation between the superfluid mass current, $\boldsymbol{p}$, and superfluid velocities, $\boldsymbol{v}$, in the hydrodynamic equations assumes a tensor character with respect to the isospin indecies (1, 2):

$$\begin{pmatrix} \boldsymbol{p}_1 \\ \boldsymbol{p}_2 \end{pmatrix} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \begin{pmatrix} \boldsymbol{v}_1 \\ \boldsymbol{v}_2 \end{pmatrix}, \qquad (1)$$

where ρ_{ik} is the superfluid density tensor, which replaces the scalar superfluid density in the Landau two-fluid model. Equation (1) shows the *entrainment effect*, i.e. the transport of the mass of both condensates by the superfluid motion of each condensate, which arises due to the interaction between the quasiparticles with different isospin projections (Andreev & Bashkin 1975, Vardanian & Sedrakian 1981). In the neutron star cores the entrainment effect leads to the appearance of electric currents of protons driven by the neutron circulation, which become a source of internal axially symmetrical magnetic fields in pulsars (see Sedrakian & Shahabasian 1991 and references therein).

Except for the early stages of pulsar evolution, the temperatures of the interiors of the stars, $T \sim 10^8$ K, are much lower than the critical temperatures for the superfluid phase transition ($T_c \sim 10^9 - 10^{10}$ K), therefore the variation of the proton gap $\Delta_1(T = 0)$ is along the density profile of the star. For typical parameters of the outer core of a a neutron star, the protons are type II superconductor (Baym, Pethick and Pines 1969). With increasing matter density, the attractive interaction between protons in the $^1S_0$ partial wave channel reduces, leading to a reducton of the proton gap. If the gap becomes sufficiently small, the coherence length of the proton condensate, ξ, can become larger than the magnetic field penetration depth λ (more precisely $\lambda/\xi \le 1/\sqrt{2}$) and the proton condensate will undergo a transition from a type II to a type I superconducting state.

The aim of this paper is the determination of the magnetic field distribution within a single neutron vortex line in the case when the proton condensate forms a type I superconductor. The present problem bears a certain resemblance to the problem of the explanation of the anomalously large magnetic flux observed in the thermo-electrical experiments on the hollow superconducting cylinders. Our calculation of the Gibbs free-energy parallels developments by Aroutyunian & Zharkov (1981) and Ginzburg & Zharkov (1993) for a hollow superconducting sample in the presence of normal currents driven around a cylindrical cavity by the temperature gradients.

The paper is organized as follows. In Section 2 we set up the basic system of equations of macroscopic superfluid

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magnetohydrodynamics for the mixture of neutron and proton condensates. In Section 3 these equations are applied to a single neutron vortex and the Gibbs free-energy is obtained for this case. Section 4 shows that the minimum of the Gibbs potential corresponds to a neutron vortex with a coaxial tube of normal protons confined within a finite radius a. The dependence of the radius of the tube and its homogeneous magnetic field on the density and the values of microscopic parameters is determined. Section 5 contains several concluding remarks.

2 THE GIBBS POTENTIAL OF A SUPERFLUID NEUTRON-PROTON MIXTURE IN A ROTATING SYSTEM

Consider the system of rotating superfluid neutron-proton mixture with a charge neutralizing background of relativistic electrons and a vanishing number of quasiparticle excitations (zero temperature limit). The kinetic part of the energy of neutron superfluid, which is by far the most dominant part of the energy of the system, is minimized by a lattice of neutron vortices. The superfluid executes a course-grained rigid body rotation with an angular velocity Ω, supported by a vortex lattice of density $n = 2\Omega/\kappa$ per unit area, where κ is the quantum of circulation. The entrainment effect does not changes this result to any considerable extent: the correction to the mass current of the neutrons is of the order of the ratio of proton to neutron density, which is typically of the order of $0.01-0.05$. At the same time, the resulting magnetic energy density is lower by orders of magnitude than the kinetic energy density. Thus, the minimization of the total energy of the system can be carried out in two steps. In the first step, one minimizes the kinetic energy of rotating neutron superfluid. Once the neutron superflow pattern is fixed by minimization of the rotational energy, in the second step the thermodynamic corrections to the energy of the system are found by the minimization of the Gibbs free-energy associated with the unentrained part of the proton condensate (the motion of the entrained part follows that of neutron superfluid and is determined at the first step of the minimization procedure). We shall concentrate on the second step of the minimization problem under assumption that the proton liquid forms a type I superconductor. A recent account of the case of type II superconductivity is given in Sedrakian & Sedrakian (1995).

Taking account of the foregoing discussions, the thermodynamical Gibbs potential for a mixture of superfluid neutrons and superconducting protons can be written as

$$\mathcal{G} = \mathcal{F} - \frac{1}{c}\int \boldsymbol{A}\cdot\boldsymbol{j}_{12}\, dV - \boldsymbol{\Omega}\times\boldsymbol{M}, \tag{2}$$

where $\boldsymbol{j}_{12}$ is the entrainment current defined below (equation 8), $\boldsymbol{A}$ is the vector potential of the magnetic field, $\boldsymbol{\Omega}$ is the angular velocity of the normal component, and the free-energy and the angular momentum of the system are

$$\begin{aligned}\mathcal{F} &= \mathcal{F}_c + \frac{1}{2}\int \left(\rho_{11}v_1^2 + 2\rho_{12}\boldsymbol{v}_1\cdot\boldsymbol{v}_2 + \rho_{22}v_2^2\right) dV \\ &+ \frac{1}{8\pi}\int B^2 dV - \frac{1}{2}\int \rho\,(\Omega\, r)^2 + \boldsymbol{\Omega}\times\boldsymbol{M},\end{aligned} \tag{3}$$

$$\boldsymbol{M} = \int \boldsymbol{r}\times(\boldsymbol{p}_1+\boldsymbol{p}_2)\; dV, \tag{4}$$

where ρ is the total density of baryonic component. The first term in equation (3), $\mathcal{F}_c$, is the superfluid condensation energy. The remaining terms are the sum of the kinetic and the magnetic energy of the system. Here the gradient invariant superfluid velocities are defined relative to their rigid body rotation value $\boldsymbol{\Omega}\times\boldsymbol{r}$

$$\boldsymbol{v}_1 = \frac{\hbar}{2m_1}\boldsymbol{\nabla}\chi_1 - \frac{e}{m_1 c}\boldsymbol{A}, \tag{5}$$

$$\boldsymbol{v}_2 = \frac{\hbar}{2m_2}\boldsymbol{\nabla}\chi_2, \tag{6}$$

where $m_{1,2}$ denotes the bare mass, $\chi_{1,2}$ the phase of the superfluid order parameter in the rotating frame; the isospin indecies 1 and 2 refer to protons and neutrons respectively. As it can be seen from the relation (1), the diagonal elements of the superfluid density matrix, ρ_{11} and ρ_{22} are the densities of the unentrained parts of the proton and neutron condensates, respectively, while the off-diagonal elements $\rho_{12}=\rho_{21}$ are the densities of entrained parts; these are related to the microscopic variables according to relations (Andreev & Bashkin 1975; Vardanian & Sedrakian 1981)

$$\rho_{12} = \frac{k}{1+k}\rho_1, \quad \rho_{11} = \frac{1}{1+k}\rho_1\,, \tag{7}$$

where in the mean-field approximation the entrainment coefficient is $k = (m_1^* - m_1)/m_1^*$, and m_1^* and m_1 are the effective and the bare mass of protons, ρ_1 is the total density of the proton condensate, and the following sum rules hold

$$\rho_2 = \rho_{22}+\rho_{12}; \quad \rho_1 = \rho_{11}+\rho_{12}; \quad \rho = \rho_1+\rho_2, \tag{8}$$

where ρ_2 is the total density of the neutron condensate.

The magnetic induction $\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A}$ is determined from the Maxwell equation,

$$\boldsymbol{\nabla}\times\boldsymbol{B} = \frac{4\pi}{c}(\boldsymbol{j}_{11}+\boldsymbol{j}_{12}), \tag{9}$$

where the proton supercurrent is split into unentrained and entrained parts, respectively,

$$\boldsymbol{j}_{11} = \frac{e}{m_1}\rho_{11}\boldsymbol{v}_1, \quad \boldsymbol{j}_{12} = \frac{e}{m_1}\rho_{12}\boldsymbol{v}_2. \tag{10}$$

On the scale of a single neutron vortex, $\boldsymbol{j}_{12}$ is the entrainment current which is generated by the neutron superfluid circulation in the neutron vortex, while $\boldsymbol{j}_{11}$ is the Meissner current appearing due to the presence of unentrained proton condensate ($\rho_{11}\neq 0$). Applying the curl operation to equation (9) one finds the London equation for the case of type I proton superconductor

$$\boldsymbol{B} + \lambda^{-2}\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{B} = k\phi_0\delta^{(2)}(\boldsymbol{r}-\boldsymbol{r}_0)\,, \tag{11}$$

where $\phi_0 = \pi\hbar c/e$ is the flux quantum and $\boldsymbol{r}_0$ is the radius vector defining the position of neutron vortex in the lattice plane and λ is the magnetic field penetration depth,

$$\lambda^2 = \frac{m_1^2c^2}{4\pi e^2\rho_{11}}.$$

Further, a change of the origin of the free-energy

$$\mathcal{F}' = \mathcal{F} - \mathcal{F}_{c2} - \frac{1}{2}\int \rho_{22}\,v_2^2\; dV + \frac{1}{2}\int \rho\;(\Omega\, r)^2\; dV, \tag{12}$$

can be made, since the neutron superfluid velocity, $\boldsymbol{v}_2$, is essentially the familiar solution of equation (6) for a single neutron vortex, while $\mathcal{F}_{c2}$ - the condensation energy of neutrons - is an additive constant. Finally, eliminating the magnetic induction in favor of the vector potential from equation (5) and denoting the condensation energy of protons by $\mathcal{F}_{c1}$, we find

$$\begin{aligned}\mathcal{F}' &= \mathcal{F}_{c1} + \int \left[\frac{1}{8\pi\lambda^2} \left(\frac{\phi_0}{2\pi} \boldsymbol{\nabla}\chi_1 - \boldsymbol{A} \right)^2 \right. \\ &+ \left. \frac{1}{c} \left(\frac{\phi_0}{2\pi} \boldsymbol{\nabla}\chi_1 - \boldsymbol{A} \right) \cdot \boldsymbol{j}_{12} + \frac{1}{8\pi} \left(\boldsymbol{\nabla} \times \boldsymbol{A} \right)^2 \right] dV. \quad (13)\end{aligned}$$

Equations (2), (9), (11) and (13) form the basic system of equations of the problem.

3 THE DISTRIBUTION OF EQUILIBRIUM MAGNETIC FIELD WITHIN A NEUTRON VORTEX

Type I superconductors in the presence of imposed currents (or equivalently magnetic field) break into successive domains of superconducting and normal regions; the geometry of the domains is determined by the symmetry of the problem. Assume that the proton superconductor goes over to the normal state within a cylinder of a radius a, concentric with the neutron vortex, forming a tube of a normal matter. (The cylindrical geometry of the tube is dictated, in our case, by the symmetry of the neutron vortex). Let the volume inclosed within the radius a be $V_<$, while that between the concentric cylinders a and b, where b is the outer radius of the neutron vortex, be $V_>$.

Because the proton liquid is normal within the volume $V_<$, the superconducting currents vanish in that region, $\boldsymbol{j}_{11} = \boldsymbol{j}_{12} = 0$. The homogeneous magnetic field $B \equiv H_{cm}$ (we neglect for simplicity the paramagnetic effects for protons in the normal state) is determined, in this region, by the continuity condition $B_z = H_{cm}$ at the boundary $r = a$. The integration of equation (2) over the volume $V_<$ is straightforward

$$\mathcal{G}^< = \frac{H_{cm}^2}{8\pi} V_<. \quad (14)$$

Decomposing the last term in equation (13) in a sum of volume and surface integrals and using equation (9), the Gibbs free-energy in the volume $V_>$ becomes

$$\begin{aligned}\mathcal{G}^> &= -\frac{H_{cm}^2}{8\pi} V_< - \alpha_1 V_> \\ &+ \int \left[\frac{\phi_0}{16\pi^2\lambda^2} \left(\frac{\phi_0}{2\pi} \boldsymbol{\nabla}\chi_1 - \boldsymbol{A} \right) \cdot \boldsymbol{\nabla}\chi_1 \right. \\ &+ \left. \frac{1}{c} \left(\frac{\phi_0}{2\pi} \boldsymbol{\nabla}\chi_1 - \boldsymbol{A} \right) \cdot \boldsymbol{j}_{12} - \frac{1}{2c} \boldsymbol{A} \cdot \boldsymbol{j}_{12} \right] dV_>, \quad (15)\end{aligned}$$

where α_1 is the condensation energy density of superconducting protons.

Let us turn to the determination of non-vanishing components of vectors $\boldsymbol{A}$, $\boldsymbol{\nabla}\chi_1$ and $\boldsymbol{j}_{12}$. The cylindrical symmetry of the problem implies that all unknown quantities are functions of the distance r from the axis of the neutron vortex and the azimuthal angle φ. From equations. (5) and (10) the azimuthal components of vectors $\boldsymbol{\nabla}\chi_1$ and $\boldsymbol{j}_{12}$ are

$$(\boldsymbol{j}_{12})_\varphi = \frac{Q}{r}, \quad (\nabla\chi_1)_\varphi = \frac{m}{r}, \quad (16)$$

where $Q = (e\hbar/2m_1m_2)\rho_{12}$ and m is an integer number; the second relation follows form the quantization of the proton supercurrent circulation around the cylindrical tube. The non-vanishing azimuthal component of the vector potential A_φ is expressed in terms of B_z component of the magnetic induction using equation (9):

$$A_\varphi = \frac{\hbar c}{e} \frac{m}{r} + \frac{4\pi\lambda^2 Q}{c} \frac{1}{r} + \lambda^2 \frac{dB_z}{dr}. \quad (17)$$

The boundary conditions for the magnetic inductions are $B_z(a) = H_{cm}$ and $B_z(b) = 0$. From the first conditon it follows, particularly, that the circulation of the vector $\boldsymbol{A}$ on the radius a is

$$\oint \boldsymbol{A} \cdot d\boldsymbol{l} = \pi a^2 H_{cm}. \quad (18)$$

and therefore $A_\varphi(a) = (a/2)\, H_{cm}$, or combining with equation (17)

$$H_{cm} = \frac{m\phi_0}{\pi a^2} + \frac{8\pi\lambda^2 Q}{ca^2} + \frac{2\lambda^2}{a} \frac{dB_z}{dr}\bigg|_{r=a}. \quad (19)$$

The determination of A_φ function is accomplished by finding the function $B_z(r)$ from equation (11); the latter is a source-free in the region $r > a$. The solution with the boundary conditions above is

$$B_z(r) = \frac{\phi_0}{\pi a^2} \left(m + \frac{8\pi^2\lambda^2}{c\phi_0} Q \right) \frac{\mathcal{N}(r)}{\mathcal{D}}, \quad (20)$$

where

$$\begin{aligned}\mathcal{N}(r) &= I_0\left(\frac{b}{\lambda}\right) K_0\left(\frac{r}{\lambda}\right) - K_0\left(\frac{b}{\lambda}\right) I_0\left(\frac{r}{\lambda}\right), \quad (21) \\ \mathcal{D} &= I_0\left(\frac{b}{\lambda}\right) K_2\left(\frac{a}{\lambda}\right) - K_0\left(\frac{b}{\lambda}\right) I_2\left(\frac{a}{\lambda}\right), \quad (22)\end{aligned}$$

and K's and I's are the modified Bessel functions. Using this result, equation (19) takes the form

$$H_{cm} = \frac{\phi_0}{\pi a^2} \left(m + \frac{8\pi\lambda^2}{\mathcal{L}\phi_0} H_e(a) \right) \frac{\mathcal{N}(a)}{\mathcal{D}}, \quad (23)$$

where

$$H_e(r) = \frac{4\pi}{c} Q\mathcal{L}, \quad \mathcal{L} = \ln\frac{b}{r}. \quad (24)$$

Here $H_e(r)$ is the magnetic field intensity created by the entrainment currents. The integrations in equation (15) using equations (16) and (23) are straightforward, and the result for total Gibbs potential $\mathcal{G} = \mathcal{G}^< + \mathcal{G}^>$ is

$$\begin{aligned}\mathcal{G} &= -\alpha_1 V_> + \frac{\phi_0^2}{8\pi^2 a^2} \left\{ \left(m - p\frac{\pi a^2 H_e(a)}{\phi_0} \right)^2 \frac{\mathcal{N}(a)}{\mathcal{D}} \right. \\ &+ \frac{4\lambda^2}{a^2\mathcal{L}} \left(\frac{\pi a^2 H_e(a)}{\phi_0} \right) \left(m - p\frac{\pi a^2 H_e(a)}{\phi_0} \right) \frac{\mathcal{N}(a)}{\mathcal{D}} \\ &+ \left. q \left(\frac{\pi a^2 H_e(a)}{\phi_0} \right)^2 \right\}, \quad (25)\end{aligned}$$

where $q = p_0 - p$ and

$$p = \frac{\mathcal{N}(a)}{\mathcal{D}} - \frac{2\lambda^2}{a^2\mathcal{L}},$$

$$p_0 = \frac{1}{\mathcal{L}^2}\left(\frac{b-a}{a}+\frac{(b-a)^2}{2a^2}-\mathcal{L}\right), \quad (26)$$

where p_0 is the limit of p for $\lambda \to \infty$. Note that equation (25) has the proper $\mathcal{G} \to 0$ behaviour when $\lambda \to \infty$ for a finite $(b-a)/b \ll 1$, i.e. of the Gibbs potential is measured from its value in the normal state (cf. Aroutyunian and Zharkov 1981).

4 THE SIZE OF THE NORMAL PROTON TUBE WITHIN THE NEUTRON VORTEX

As may be seen from the expression (25), which contains two unknown parameters m and a, the increase in the energy of the system associated with the magnetic field is minimal if one chooses

$$m = p\frac{\pi a^2 H_e(a)}{\phi_0}. \quad (27)$$

Note that for the present problem (in contrast to the case considered in Aroutyunian and Zharkov 1981) there are no energetic barriers for the fulfillment of this condition, since for the superconducting protons in the n-p-e phase of neutron stars hysteresis effects are absent. Imposing the condition (27) we find from equations (23) and (25), respectively,

$$H_e(a) = H_{cm}, \quad \mathcal{G} = -\alpha_1 V_> + q\pi a^2\frac{H_{cm}^2}{8\pi}. \quad (28)$$

Note that the first condition coincides with the requirement $\boldsymbol{j}_{12} = 0$. Using the definition $H_{cm}^2/8\pi = \alpha_1$ one finds

$$H_{cm} = H_t(a) = 2\sqrt{2\pi\alpha_1}. \quad (29)$$

The relation determining the radius a follows by combining this expression with equation (24):

$$a = b\ \exp\left(-\frac{2\sqrt{2\pi\alpha_1}}{H_0}\right), \quad (30)$$

where

$$H_0 = \frac{k\phi_0}{2\pi\lambda^2}, \quad \alpha_1 = \frac{\nu(\epsilon_F)\Delta_1^2}{4}, \quad \nu = \frac{3^{1/3}\, m_1^{*\,2/3}}{\pi^{4/3}\,\hbar^2}\,\rho_1^{1/3}. \quad (31)$$

Here $\nu(\epsilon_F)$ is the density of states of protons on the Fermi-surface.

Table 1 gives the values of microscopic parameters of the proton superconductor for several values of total baryonic density. The dependence of the proton gap and the effective masses on the proton Fermi wave-vector, k_{F1}, are from Baldo *et al* (1992). The relation between k_{F1} and the total baryonic density corresponds to the equation of state of Wiringa, Fiks & Fabrocini (1988). The last column is the radius of the normal region following from equation (30) for the neutron intervortex distance $b = 4 \times 10^{-3}$ cm, corresponding to the equilibrium rotation frequency of the Vela pulsar $\Omega = 70.6$ s^{-1}. If the proton gap eventually closes with increasing density, the radius of the normal region becomes b, i.e. the protons go over in the normal state in the whole volume of the vortex.

5 CONCLUDING REMARKS

The present paper analyses the implications of a transition of a proton superconductor from type II to type I superconducting state for the magnetic structure of neutron vortex lattice in the core of the neutron star, taking into account the entrainment effect. The main result is that, owing to the entrainment effect, the type I proton superconductor breaks into successive normal superconducting regions. We analyzed the Gibbs potential in the case in which normal regions are of cylindrical shape, which is most naturally implied by the symmetry of the problem. We do not exclude the possibility, however, that the system may support a more complicated pattern of normal-superconducting domains, and those should be examined in future.

The transition from the type I to the type II superconducting state occurs at densities $\sim 3\ \rho_{sat}$. These densities are indeed achieved in neutron star models with canonical masses $\sim 1.4\ M_\odot$ which are based on moderately-soft equations of states (e. g. Wiringa et al. 1988). Though the density range occupied by type I proton superconductor is not as broad as for a type II superconductor, the density profile of the star at high densities is commonly quite flat, implying that the type I proton superconductor can occupy a considerable fraction of the volume of a superconducting core. The magnetic field moment, associated with the neutron vortex lattice, would be aligned with the rotation vector unless the star is a non-axisymmetrical body at the instance of the nucleation of superconducting phase. Since this would require rotational velocities close to the mass-shedding limit, the nucleation of non-axisymmetrical field via this mechanism dose not appear to be a relistic possibility. The mean magnetic induction of a single neutron vortex is $\langle B\rangle \simeq H_e(a)\,(a/b)^2 \sim 10^{13} - 10^{14}$ G for densities listed in the table; it drops, however, to zero in the limit $a \to b$ when the entrainment currents vanish.

Present results on the ground state configuration of the type I proton superconductor can be used to address the problems of dissipative motion of neutron vortex lattice, resulting in a effective coupling of the neutron superfluid to the normal electron liquid, heat generation via irreversible processes and other related problems.

Finally, we note that a situation similar to that discussed in the present paper can be obtained in the metallic type I superconductors of cylindrical shape when a temperature gradient is applied between the axis and the boundary of the sample. In principle, the effect of transition of the central part of the cylinder to the normal state should be observable in this type of experiments.

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Table 1. Parameter values and the size of the normal region in type I proton superconductor

ρ ($\times 10^{14}$ g cm^{-3})	k_{F1} (fm^{-1})	m_1^*/m_1	Δ_1 (MeV)	ξ (fm)	λ (fm)	H_0 ($\times 10^{14}$ G)	a/b
7.91	0.85	0.69	0.3	54.2	41.58	5.9	0.14
8.30	0.88	0.68	0.2	97.0	39.20	6.8	0.32
8.56	0.90	0.68	0.1	197.1	37.87	7.4	0.59

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